432 Park Ave Supplemental Material

Brief Background

- Construction completed in 2015
  - Total cost estimated between 1.25 and 3.12 billion dollars
- Tops off at 1,396 feet tall (96 floors)
  - 5^{th} tallest building in New York
  - 26^{th} tallest building in the world
  - (Although these statistics seem to change regularly so they might be out of date in the next couple of weeks!)
- One of the most slender buildings in the world
  - 15:1 height-to-width ratio requires a very unique structure to keep the building from swaying too much
  - Required approval from the FAA
- NY Times Article – expose on 432 Park Ave.

Two-Degree-of-Freedom Model

- There are two equations that describe the motion of each degree of freedom
- This model returns 2 coupled ODEs
- This model will have two modes. Each of the two modes represent the natural frequencies of the system.

Multiple-Degree-of-Freedom Model

- Has N equations that describe the motion of each degree of freedom and returns N coupled ODEs. This also means there will be N modes. For our building we have 96 equations of motion and 96 modes. Quite a few. The eigenvalues of the system describe the modes, or the natural frequencies of the system. Each eigenvector describes the systems motion for each eigenvalue.
- For our approximation we idealize the structure as a coupled spring-mass system
  - Each floor is assumed to be a separate mass. For our structure, we have variables mass such that floors 1-85 have a lower mass (487,350 kg) and floors 86-96 have a higher mass (896,716.8 kg). We also estimate the mass of each floor based on each floors volume (For floors 1-85: 203.0625 m^3 and for floors 86-96: 373.6875 m^3) multiplied by the density of constituent (2400 kg/m^3) material. This returns the weight of the floor.

- Support beams serve as springs in series. Thus, we can find the spring constant for one support and multiply the spring constant by the number of beams per floor to find the total spring constant for a particular floor. We approximate that the spring constant is given by:

  $k = 48\frac{\mathrm{E*I}}{L^3}$ where E is Young's modulus, I is the area moment of inertia, and L is the length of each beam

  - Young's Modulus is a known value. For high strength compression concrete, it is 30 Giga Pa (conversion was necessary for calculations, but that was the displayed value)
  - The area moment of inertia for our structure was estimated to be $I = \frac{1}{12}ab^3$. a is the width of the beam, and b is the depth of the beam. Orientation is crucial here because b, the depth, is defined to be the larger value. We found I=.1692 m^4
  - Finally, we found the length of each beam to be 4.72 m. Note: this is the *full* length of the beam since our equations approximate that the beam is embedded in the slab of the above floor—not simply the exposed length of the beam from one floor to the next. This building was designed to have particularly high ceilings for the intended affluent residents.
  - We found that there are 24 beams per floor. As mentioned above, we assume that for each floor, the spring constant of a floor is K*n where n is the number of beams per floor. Simply, we assume the springs are in series.
  - With all of these values known, we estimate the spring constant of floors 1-85 to be 5.56E10 N/m. Using a relatively reasonable guess of a .78k change in the spring constant for the upper floors, 86-96, we estimate the second spring constant to be 5.33E10. As the mass changes with these floors, so to must the spring constant.

- Casting this as a matrix allows us to calculate (through MATLAB in our case) the modes and mode shapes of a structure by finding the eigenvectors and eigenvalues of the structure.
  - This describes the building's movement and behavior where the eigenvalues are the modes (natural frequencies) of the building and the eigenvectors describe the motion (mode shapes) of the building for a given eigenvalue. Each eigenvalue has a corresponding eigenvector. We found these values from our MATLAB script as mentioned below.

MATLAB

- We input the mass matrix and the spring matrix into our MATLAB code to solve for the 96 eigenvalues and eigenvectors. Our solution to this can be found in the code. We studied the first ten modes as those tend to be nearer to the frequencies that occur naturally (earthquakes, hurricane winds, etc.) and thus, could impact the building's structural integrity. For example, earthquakes tend to occur between 0.2 and 10 Hz. However, for specific areas, past frequencies of earthquakes have been recorded and potential future frequencies of earthquakes can be deduced from these findings. Thus, structural engineers can avoid constructing buildings with a natural frequency similar to the frequency of earthquakes experienced in those areas.

Tuned Dampers

- A tuned mass damper is an oscillating object that moves out of phase with the motion of the building to which it is attached. By oscillating out of phase with the building, the TMD counteracts the resonant vibration of the building and dissipates its energy. The mass should oscillate orthogonal to the building's motion but must be tuned to do so.
- This building has two ton tuned mass dampers (combined weight of 1,370 tonnes) that oscillate perpendicular to the buildings motion. This minimizes sway and ensures that building isn't oscillating at low frequencies which often causes nausea and sickness for humans.
  - Also, there are several "gap floors" with no mass which helps ensure that wind can pass through the building, further reducing sway. Because of issues inserting these gaps into the mass matrix, we have neglected them and acknowledge that our first order approximation is certainly not the most accurate accounting of the building's structural makeup! Moreover, outriggers bind the exterior and interior supports to create more rigidity in the structure. The relationship between the outriggers and the estimated spring matrix is unclear, so we have not included this in our calculations (but it is nevertheless an interesting design to mention).