A SIR Approach to Analyzing the Spread of the Current Novel Coronavirus (2019-nCoV) Outbreak in Hong Kong

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April 21, 2020

1 Introduction

The standard SIR model (Susceptible, Infected, Recovered) has been used for decades to study a variety of different systems. It has been used to model the predator-prey relationship between wolves and deer in the northeast and to explain the spread of rumours in a large population. One of the canonical uses for the SIR model is to model the spread of diseases. This paper will discuss the SIR model and its applications for analyzing the spread of the current novel coronavirus (2019-nCoV) in the city of Hong Kong. The current novel coronavirus has been spreading rapidly through Wuhan, China in the past few weeks and months. Common signs of infection include respiratory symptoms, fever, cough, shortness of breath and breathing difficulties. In more severe cases, infection can cause pneumonia, severe acute respiratory syndrome, kidney failure and even death. Our model will look at the effects of this virus on a large city that has relatively few cases reported thus far. As of today, Hong Kong has reported sixty two coronavirus cases with 1 death. We will look at the spread of the virus due to those sixty two individuals that were infected.

2 Methods

2.1 The Standard S.I.R Model

The standard S.I.R model comes from Biology and is a simple yet powerful way of describing the spreading of an infectious disease. The model consists of three ordinary first order differential equations that describe the change in population of three different types of people; Susceptible (those that can get infected), Infected (those who are currently infected with the disease), and Recovered (those who had the disease but recovered from it and are now considered immune). The equations used to model this relationship in a population of N people are:

$$N = S(t) + I(t) + R(t) \tag{1}$$

$$\frac{dS(t)}{dt} = -\alpha S(t) \tag{2}$$

$$\frac{dI(t)}{dt} = \alpha S(t) - \beta I(t) \tag{3}$$

$$\frac{dR(t)}{dt} = \beta I(t) \tag{4}$$

With α equal to the rate of infection and β equal to the rate of recovery. This simple model has been modified in various ways to improve the accuracy of its results. Correlations can be built in, death rates accounted for, and the idea of a limited time of immunity can be integrated into this versatile model.

2.2 Our Modification of the S.I.R Model

In order to study the spread of 2019-nCoV in Hong Kong, we needed to modify the system ordinary differential equations displayed above to account for the unique situation of a highly infectious disease spreading through a largely populated area. Our modified equations are based on those used by Joseph T Wu et al. in their forecast of the potential domestic and international spread of the 2019-nCoV(**?**):

$$N = S(t) + E(t) + I(t) + R(t) \tag{5}$$

$$\frac{dS(t)}{dt} = -\frac{S(t)}{N}\frac{2}{\alpha}E(t) + \gamma - \frac{\gamma}{N}S(t) \tag{6}$$

$$\frac{dE(t)}{dt} = \frac{S(t)}{N}\frac{2}{\alpha}E(t) - \frac{E(t)}{\beta} - \frac{\gamma}{N}E(t) \tag{7}$$

$$\frac{dI(t)}{dt} = \frac{E(t)}{\beta} - \frac{I(t)}{\alpha} - \frac{\gamma}{N}I(t) \tag{8}$$

$$\frac{dR(t)}{dt} = -\Delta + \frac{\gamma}{N}(S(t) + E(t) + I(t)) + \frac{I(t)}{\alpha} \tag{9}$$

Here, N is the total population of Hong Kong, $S(t)$ is the rate of susceptible individuals, $E(t)$ is the rate of latent individuals, $I(t)$ is the rate of infectious individuals, and $R(t)$ is the rate of recovered individuals. Moreover, we assume the basic reproductive number to be 2 (determined from rates of previous, similar viruses [SARS, MERS]), α is the infectious period, which is assumed to be 10 days, and β is the incubation period, which is assumed to be the same as the infectious period. Finally, γ was the number of inbound travelers per day and Δ was the number of outbound travelers per day.

We took these modified equations and analyzed them using Python's odeint feature and a modified S.I.R coded provided by The University of Oregon.

3 Results

3.1 Analysis

Through modeling the rate of change for the susceptibility, latency, infection, and removal for 2019-nCoV, we were able to determine how the disease will progress in Hong Kong for several different sets of initial conditions. The pattern of the spread of the virus remains the same over several different sets of initial conditions. This model is useful in helping to determine when peak infection will occur in different areas of the country, and can be modified to account for different parameters in different cities. This model will benefit areas of high air traffic, in order to stop the spread of 2019-nCoV with as few casualties as possible.

3.2 Graph Results

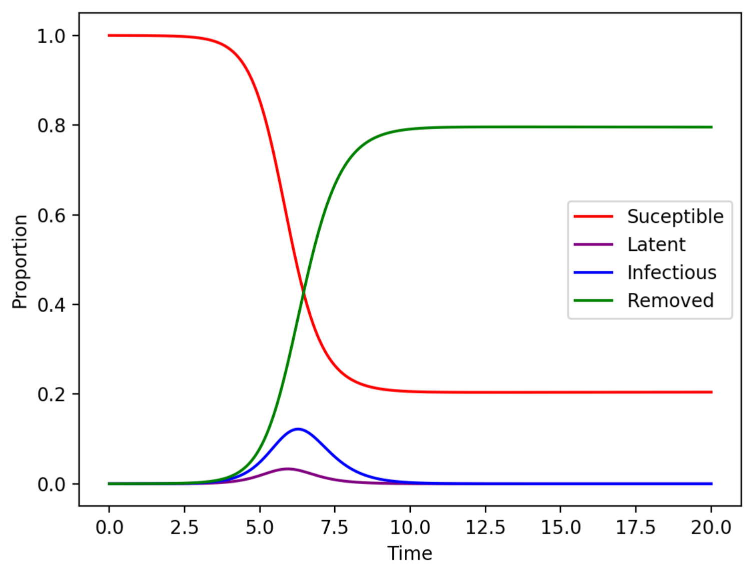

Figure 1: Results for: $S(t) = 7.4$ million, $E(t) = 53$, $I(t) = 53$, and $I(t) = 2$. $\alpha = 0.4$, $\beta = 0.1$, $\gamma = 1,000$, and $\delta = 70$

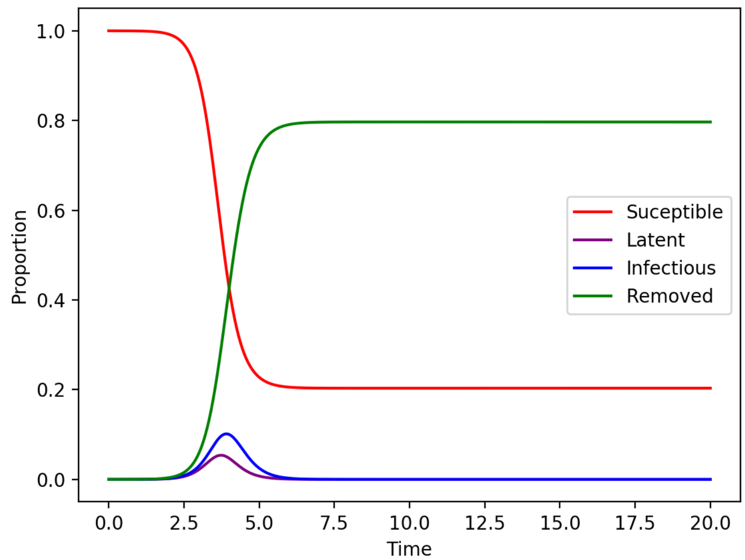

Figure 2: Results for: $S(t) = 7.4$ million, $E(t) = 53$, $I(t) = 53$, and $I(t) = 2$. $\alpha = 0.2$, $\beta = 0.1$, $\gamma = 100$, and $\delta = 7,000$

4 COVID-19 Update (April 2020)

Once again, we face the problem of not having accurate data collection in Hong Kong. However, on April 19, 2020 it was reported that Hong Kong had 1,030 confirmed cases, 602 recovered individuals, and 4 deaths.

Our biggest error is likely the proportion of the population that are latent carriers. Based on recent reports, this proportion should be quite large. In fact, these findings have been a major factor in countries instituting strict social distancing orders.

In order to refine our model, we would want a more accurate picture of Hong Kong's daily cases throughout the pandemic. With this information, we would be able to more accurately determine the coefficient of transmission as well as the latency period associated with the virus. As more and more data comes out surrounding COVID-19, it's becoming clear that the speed and magnitude of countermeasures taken by a country has had a great impact on the peak cases and level of spread in that country. For a more complete model of the situation, one should take into account these additional factors.

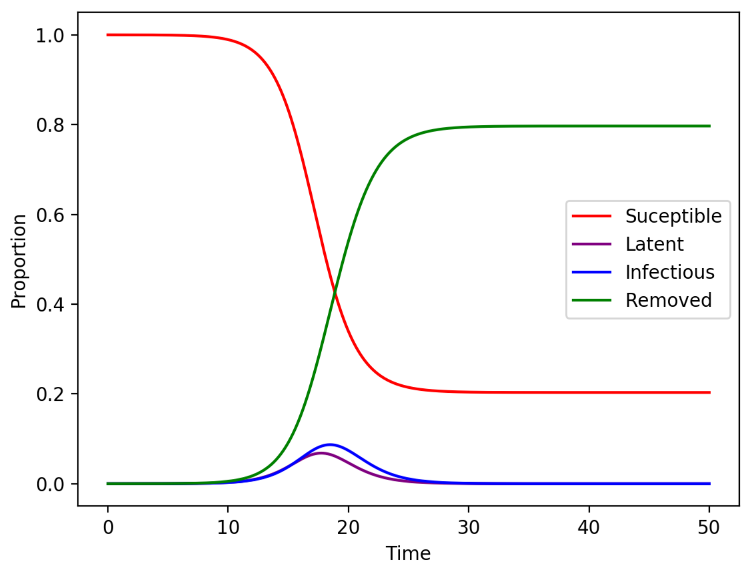

Figure 3: Results for: $S(t) = 7.4$ million, $E(t) = 53$, $I(t) = 53$, and $I(t) = 2$. $\alpha = 0.8$, $\beta = 0.6$, $\gamma = 0$, and $\delta = 50,000$