Regularized Least Square Reverse Time Migration with Prior Model

Nick Dorogy, and Reinaldo Sabbagh, Colorado School Mines

SUMMARY

For this project, we have incorporated a prior model in least square reverse time migration (LSRTM) to investigate how adding a prior model can improve or distort the seismic image. In exploration, a prior model can be used to incorporate well data or geological information that provide further constraints for subsurface reflector locations. We compared regularized LSRTM (RLSRTM) with a prior model against the standard LSRTM solution. We tested scenarios in which the prior model was both accurate and inaccurate in terms of positioning, dimensions, and confidence. When the prior model was accurate, the RLSRTM solution significantly improved the subsurface reflectivity when compared to LSRTM. However, an inaccurate prior model can distort the image, leading to a reflectivity model that misrepresents or smears the true subsurface anomaly.

INTRODUCTION

Least-squares reverse time migration offers a significant advantage over traditional methods in that it produces high-resolution images while preserving amplitude information. This is achieved by formulating the imaging problem as a least-squares minimization. However, the inherent ill-posedness of the inverse problem can lead to convergence issues. Regularization techniques and the incorporation of a prior model can significantly improve gradient-based optimization and address these challenges (Li et al. (2015)).

Field exploration often yields valuable prior information from well data and geological studies that can be included in the inversion process to improve image sharpness and reduce computation time. This project investigates the impact of a prior model, where we explore how different types of prior models can affect the final image quality.

Additionally, we compare the effects of different prior models, highlighting the importance of integrating diverse data types that may constrain inaccurate results. This comparison showcases how an inaccurate prior model can negatively impact the final image.

Numerical tests were conducted using Devito (R. A. de Cristo and Pestana (2021)) to demonstrate the changes produced by RLSRTM with different prior model information. Devito is a finite difference solver with symbolic programming that generates a c-optimized code more efficient for large scale finite difference models.

THEORY

Least square migration is based on the assumption that, a linear operator $\mathbf{L}$, based on the wave equation, acts on a reflectivity model, $\mathbf{m}$, to produce data, $\mathbf{d}$:

$$\mathbf{Lm} = \mathbf{d}$$

The migration operator is therefore defined as the the adjoint of $\mathbf{L}$ and transforms the data to the model domain (e.g. a seismic image):

$$\mathbf{L^T d} = \mathbf{m}$$

LSM aims to find the image that best predicts, in a least-squares sense, the recorded seismic data. To accomplish this, we minimize the following function:

$$\phi = \frac{1}{2}\|\mathbf{Lm} - \mathbf{d}\|_2^2$$

whose gradient can be expressed as:

$$\nabla\phi = \mathbf{g} = \mathbf{L}^T[\mathbf{Lm} - \mathbf{d}]$$

If we wish to incorporate prior information into our solution, the regularized LSM must now be expressed as:

$$\phi = \frac{1}{2}\|\mathbf{Lm} - \mathbf{d}\|_2^2 + \lambda\|\mathbf{m} - \mathbf{m}_{prior}\|_2^2$$

Now, the gradient of the regularized objective function becomes:

$$\nabla\phi = \mathbf{g} = \mathbf{L}^T[\mathbf{Lm} - \mathbf{d}] + \lambda[\mathbf{m} - \mathbf{m}_{prior}]$$

In the exploration setting, we might construct the prior model term with well data and stratigraphic tops interpretations, for example. For this project, we have assumed knowledge of the approximate location of the reflectivity boundaries when constructing our reference model. We then produce the LSRTM and RLSRTM solutions for each reference model example and compare their resulting images in terms of the reflector's sharpness and positioning.

The least square migration problem is solved iteratively. For each iteration we need a search direction and a step length. The model update is presented by the following equation:

$$\mathbf{m}^{k+1} = \mathbf{m}^k + \alpha\mathbf{v}^k$$

where the model at iteration $\mathbf{m}^{k+1}$ is equal to the model at the previous iteration $\mathbf{m}^k$ plus α which is the step length and $\mathbf{v}^k$ is the search direction. For steepest descent the search direction $\mathbf{v}$ is the gradient of the objective function. For this study we selected the step length as (A. Oliveira and dos Santos (2016)):

$$\alpha = \frac{\mathbf{g}^T\mathbf{g}_k}{\mathbf{L}\mathbf{g}_k^T\mathbf{L}\mathbf{g}_k}$$

RESULTS & DISCUSSION

For creating data in forward modeling, each iteration used 15 shots located every 70 meters at the surface. Receivers were also located at the surface and separated by a distance of 10 meters. To solve the inverse problem we computed 10 iterations in total. In Figures 1-7, the first plot shows the unique reference model used in that case study. The second and third plots show the results of the LSRTM and RLSTRTM solutions. This is followed by the true reflectivity model. The final image compares the LSRTM and RLSRTM solutions reflector intensities against that of the true model.

The regularization parameter plays an important role when finding the optimal model to the regularize square migration problem. A small regularization parameter λ values makes the contribution of the regularization and the prior model negligible. On the other hand, a high λ create the opposite results, the regularization term and the reference model will overtake the solution and the model obtained will be very close to the reference model. We tested several regularization parameters to better understand their effects on the RLSRTM solution. We first needed to select an appropriate regularization term, and in doing so, we observed that a high value was required for the regularization term. In this project, we are not using the weight matrices for the data or model ($\mathbf{W_d}$ and $\mathbf{W_m}$). These matrices serve as normalization for the data and model terms; for this reason, we must choose a high beta value to ensure that the contribution of the reference model is non-negligible.

Then, assuming centering and dimensions of the anomaly are consistent with the true model, we vary Gaussian smoothing of the reference model under three different conditions: high smoothing in which $\sigma_x = \sigma_y = 7.5$ (Figure 2), average smoothing in which $\sigma_x = \sigma_y = 5$ (Figure 1), and low smoothing in which $\sigma_x = \sigma_y = 2.5$ (Figure 3). The smoothing of the reference model represents the certainty of the prior information in the location of the reflectivity boundaries. A low smoothing represents the case where we have high confident on the prior information. As anticipated, low smoothing with the highly accurate reference model produces a sharp, clear reflectivity. When comparing the RLSRTM and LSRTM solutions, we observe that as uncertainty in the anomaly increases, the RLSRTM solution worsens. For the highly smoothed model, it is difficult to discern the anomaly at all.

We also vary the center and dimensions of the anomaly to mimic a misplaced or missized geologic feature. We begin by displacing the anomaly 1 km to the left as shown in Figure 4. Notice now that the reflector in the RLSRTM has been smeared laterally to the left due to the influence of the reference model. However, the true anomaly is still observable in its correct position. If we continue increasing our regularization term, the reference model dominates as expected. Likewise, in Figure 5, the reference model has misplaced the anomaly 10 km above its true position. We again notice the subsequent smearing but still image the true anomaly as well.

Finally, in Figure 6 and Figure 7 we expand and shrink the reference model size by 4x while maintaining the correct centering. Once again, we observe distortions and higher amplitudes in the sensitivity kernel while still imaging the true anomaly.

CONCLUSION

This analysis suggests that high confidence prior knowledge can produce a sharper image in RLSRTM than can acquired with LSRTM. If the reference model is significantly misaligned or missized however, the LSRTM approach can produce a better result. Even with an inaccurate reference model, the RLRSTM solution was still able to identify the true anomaly but artifacts introduced by the reference model can make it difficult to identify the correct boundaries.

This work highlights the importance of incorporating prior information into the inversion procedure and shows that misguided reference models produce inaccurate images.

ACKNOWLEDGMENTS

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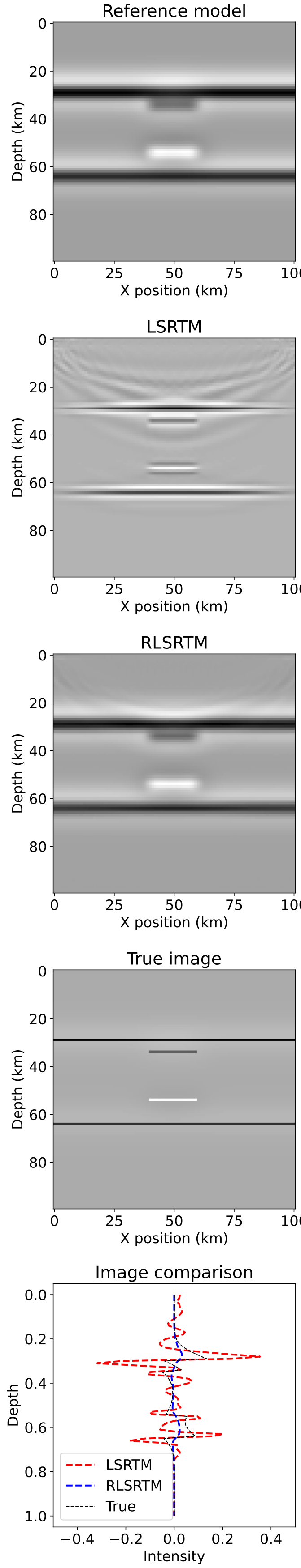

Figure 1: Reference model with center and dimensions that match the true, but with Gaussian **low** smoothing applied ($\sigma_x = \sigma_y = 2.5$).

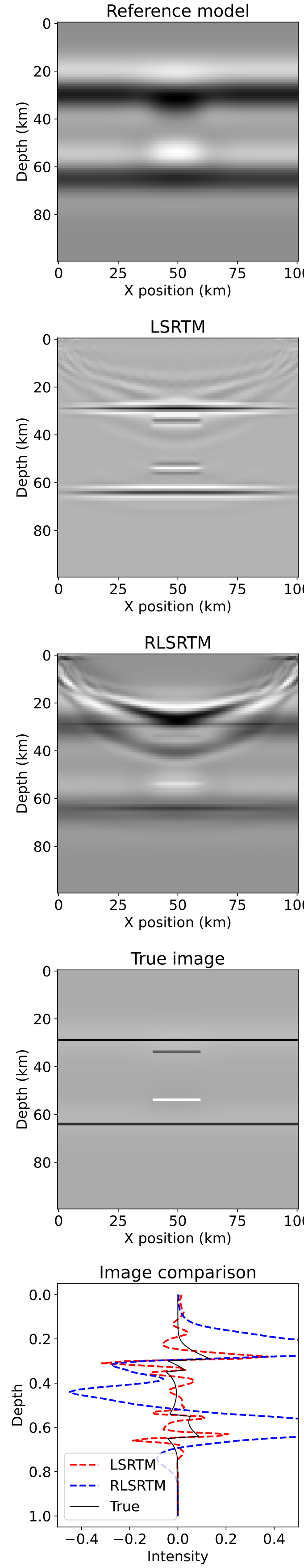

Figure 2: Reference model with center and dimensions that match the true model, but with Gaussian **average** smoothing applied ($\sigma_x = \sigma_y = 5$).

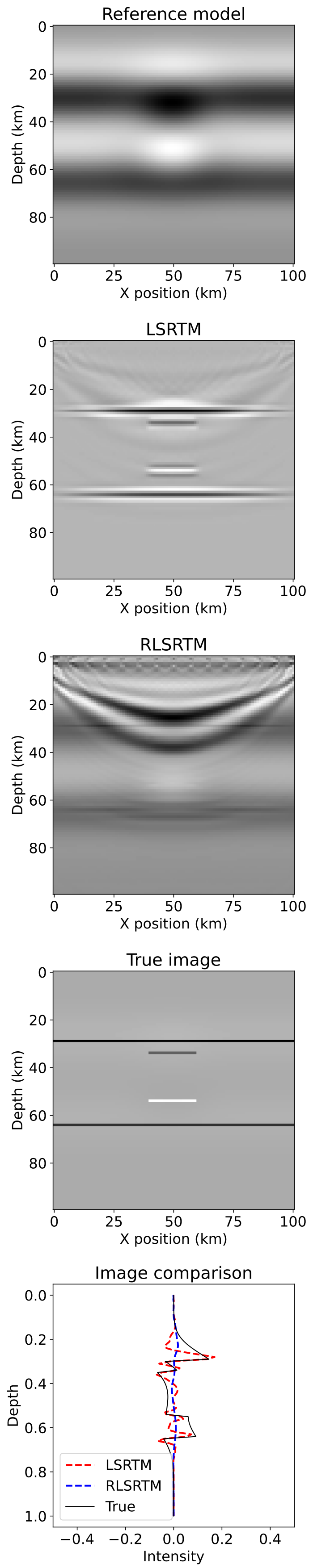

Figure 3: Reference model with center and dimensions that match the true model, but with Gaussian **high** smoothing applied ($\sigma_x = \sigma_y = 7.5$).

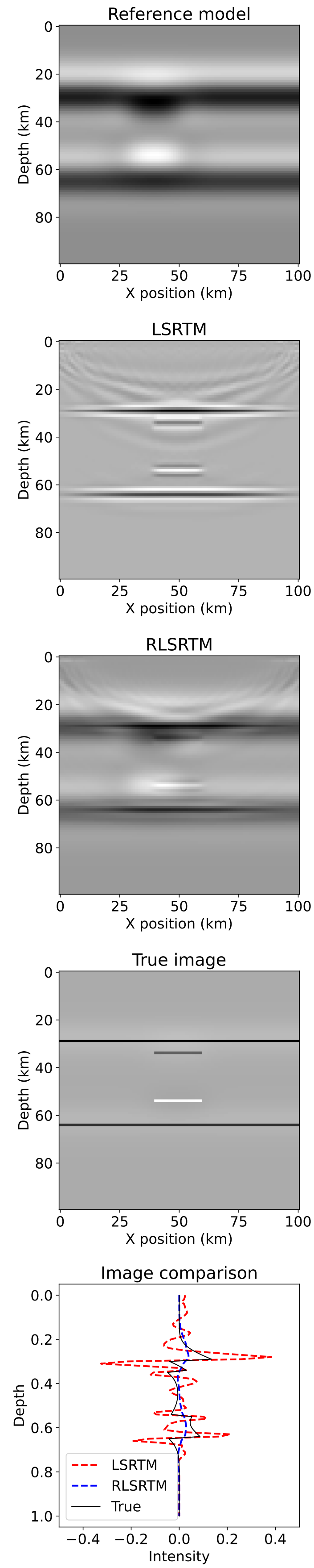

Figure 4: Reference model dimensions match the true model but the center is offset to the left by 10km. This model utilizes average smoothing ($\sigma_x = \sigma_y = 5$).

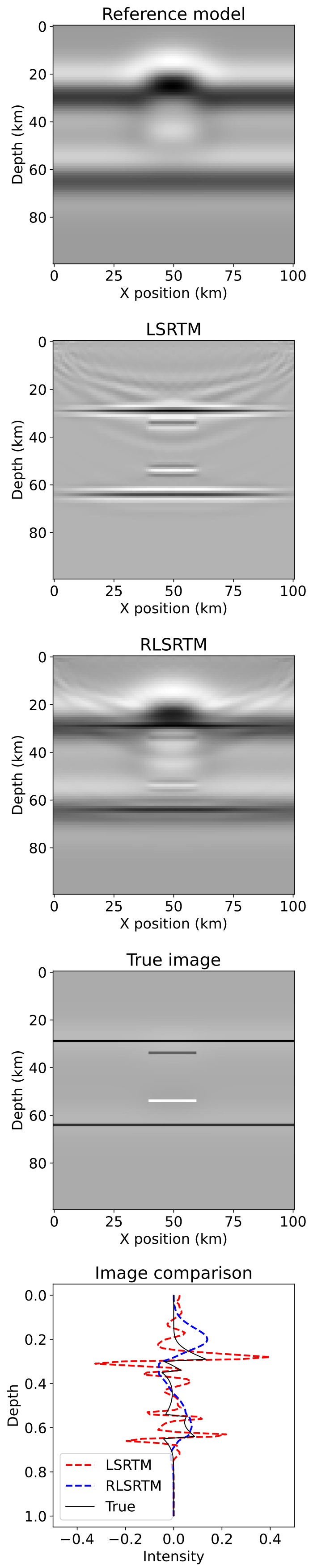

Figure 5: Reference model dimensions match the true model but the center is offset upwards by 10km. This model utilizes average smoothing ($\sigma_x = \sigma_y = 5$).

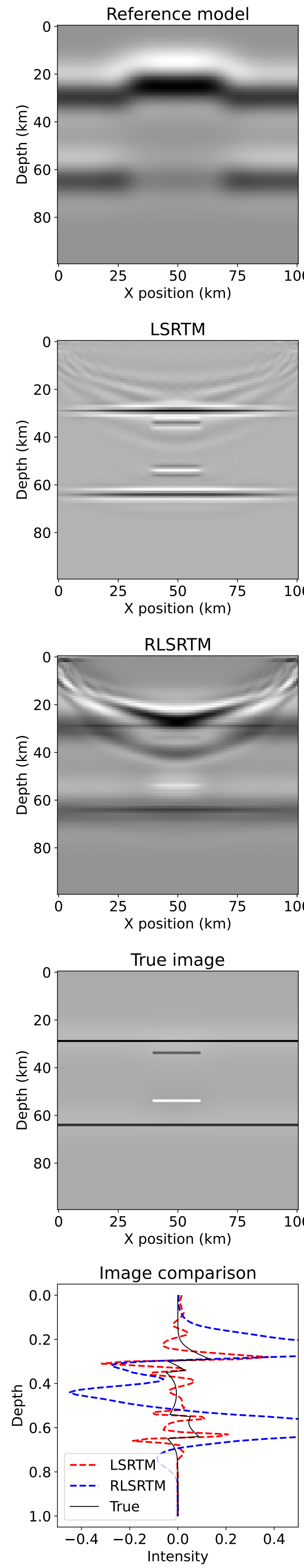

Figure 6: Reference model centered in the same location as the true model but is 4x larger. This model utilizes average smoothing ($\sigma_x = \sigma_y = 5$).

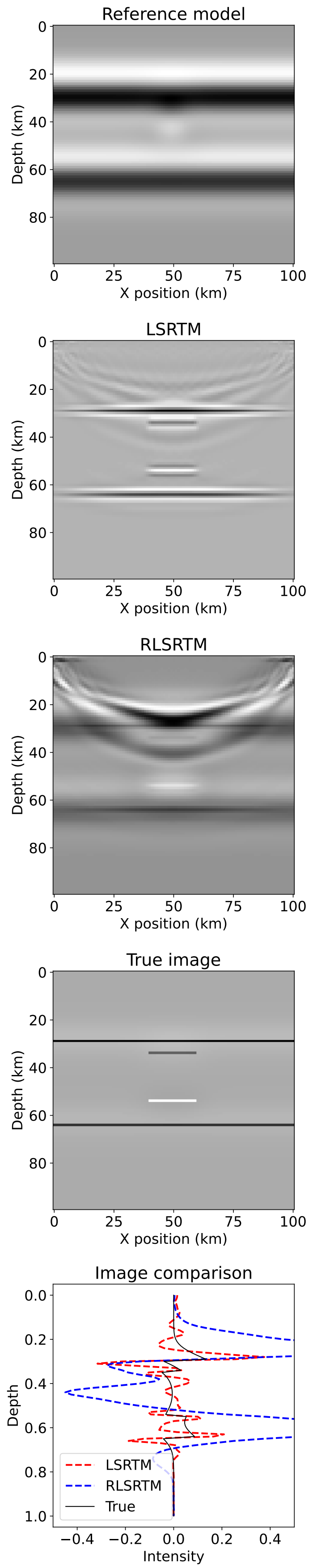

Figure 7: Reference model centered in the same location as the true model but is $4x$ smaller. This model utilizes average smoothing ($\sigma_x = \sigma_y = 5$).

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